

Tensor power spectrum with holonomy corrections in LQC

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In this paper we consider tensor perturbations produced at a bounce phase in presence of the holonomy corrections. Here bounce phase and holonomy corrections originate from Loop Quantum Cosmology. We re-derive formulas for the corrections for the model with a scalar field content. Background dynamics with a free scalar field and multi-fluid potential are considered. Since the considerations are semi-classical effects of quantum fluctuations of the background dynamics are not taken into account. Quantum and classical back-reaction effects are also neglected. To find spectrum of the gravitational waves both analytical approximations as well as numerical investigations were performed. We have found analytical solutions on super-horizontal and sub-horizontal regimes and derived corresponding tensor power spectra. Also occupation number $n_{\mathbf{k}}$ and parameter Ω_{gw} were derived in sub-horizontal limit, leading to its extremely low present value. Final results are numerical power spectra of the gravitational waves produced in presence of quantum holonomy corrections. The obtained spectrum has two UV and IR branches where $\mathcal{P}_T \propto k^2$, however with the different prefactors. Spectrum connecting these regions is in the form of oscillations. We have found good agreement between numerical spectrum and this obtained from the analytical model. Obtained spectrum can be directly applied as initial conditions for the inflationary modes. We mention possible resulting observational features of the CMB in particular B-type polarization.

I. INTRODUCTION

In the Minkowski background free gravitational waves fulfil the wave equation $(\partial_t^2 - \nabla^2)h_{\mu\nu} = 0$. Solutions of this equation are plane waves $h_{\mu\nu} = P_{\mu\nu}e^{i(\mathbf{k}\cdot\mathbf{x}-|\mathbf{k}|t)}$, here $P_{\mu\nu}$ is polarization tensor. However when cosmological expansion is turned on (we assume flat FRW background here) additional term appears and the equation of motion is modified to $(\partial_t^2 + 3H\partial_t - \nabla^2)h_{\mu\nu} = 0$, where H is Hubble parameter. We see that cosmological term acts as effective friction. When Universe undergoes expansion then $H > 0$ and gravitational waves are damped. This situation corresponds to the present stage of the evolution. However when Universe is in contracting phase $H < 0$ friction term becomes negative and gravitational waves are amplified. Such a phase of contraction is general prediction of Loop Quantum Cosmology (LQC)[1]. Contracting phase appears also in the string theory based theories of the Universe [2, 3] and many others. However in the present paper we concentrate on the LQC models where contracting and expanding regimes are joined by the phase of bounce [4]. In the last years perturbations during the bounce phase were studied extensively. Recent review on this issue can be found in [5]. In particular perturbations in the Quintom Bounce were studied in [6, 7].

In LQC many physical results can be traced in semi-classical approximation. In particular dynamics of the Universe can be recovered from the quantum corrected Friedmann equation [8, 9]. Similar approach can be also applied to describe quantum gravity effects on perturbations [10], in particular to gravitational waves. It is however worth to stress that such an approach is rather

heuristic and results obtained have to be verified by the purely quantum considerations. In particular it has not been proved yet whether phase of bounce is generally realized for the inhomogeneous loop cosmologies. However some recent studies show that in case of loop quantized inhomogeneous Gowdy spacetime, singularity is avoided [11]. In our approach inhomogeneities are treated perturbatively and we neglect their back-reaction on the background dynamics. However in the more detailed studies these effects have to be also taken into account. In the semi-classical approach applied here, quantum gravity effects are introduced by the corrections to the classical equations of motion. For tensor modes in LQC these effects were preliminary studied in Ref. [12, 13]. Later improved approach was developed [14] introducing holonomy corrections. Results of this paper are a backbone of our investigations. In this paper we assume that these corrections are valid during the whole evolution. Some preliminary studies of influence of the holonomy corrections for the gravitational waves production have been done [15, 16, 17]. However in that papers effects of the corrections to the source term were neglected. While in classical approach this term vanishes (within linear regime) in quantum regime it does contribute. In the present paper we improve these studies including a source term.

Besides the holonomy corrections also inverse volume corrections are predicted in the framework of LQC. Effects of inverse volume corrections on gravitational waves were recently studied in Ref. [18, 19]. However in the flat FRW background inverse volume corrections exhibit fiducial cell dependence. This makes those effects harder to interpret. However, in the curved backgrounds this problem disappears. Since holonomy and inverse volume effects differ qualitatively, they should be studied separately. In this paper we follow this line of reasoning. We consider consistent model where holonomy corrections in-

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fluence both background and perturbations parts.

The organisation of the text is the following. In section II we introduce the equation for tensor modes with holonomy corrections. Then in section III we define background dynamics. We consider both the model with free scalar field and with multi-fluid potential. Subsequently in section IV and section V we investigate analytically and numerically the evolution of the tensor modes. Effects of holonomy corrections are investigated. With use of numerical computations we calculate power spectra of the gravitational waves. In section VI we summarize the results. Finally in Appendix we introduce gravitational waves in LQC framework, derive particular form of the holonomy corrections and explain the employed notation.

II. GRAVITATIONAL WAVES WITH HOLONOMY CORRECTIONS

Equation for tensor modes with LQC holonomy corrections derived in [14] is given by

$$\frac{d^2}{d\eta^2} h_a^i + 2\bar{k} \frac{d}{d\eta} h_a^i - \nabla^2 h_a^i + T_Q h_a^i = 16\pi G \Pi_{Qa}^i, \quad (1)$$

where

$$T_Q = -2 \left(\frac{\bar{p}}{\bar{\mu}} \frac{\partial \bar{\mu}}{\partial \bar{p}} \right) \bar{\mu}^2 \gamma^2 \left(\frac{\sin \bar{\mu} \gamma \bar{k}}{\bar{\mu} \gamma} \right)^4, \quad (2)$$

$$\Pi_{Qa}^i = \left[\frac{1}{3V_0} \frac{\partial \bar{H}_m}{\partial \bar{p}} \left(\frac{\delta E_j^c \delta_a^j \delta_c^i}{\bar{p}} \right) \cos 2\bar{\mu} \gamma \bar{k} + \frac{\delta H_m}{\delta (\delta E_i^a)} \right] \quad (3)$$

For details and explanation of the employed notation we send to Appendix. To derive specific form of the functions T_Q and Π_{Qa}^i , matter content must be defined. In this paper we consider models with a scalar field. We consider both free and self-interacting fields. In that case matter Hamiltonian is up to the second order

$$H_m = \bar{H}_m + \frac{1}{4} \int_{\Sigma} d^3x \frac{\bar{N}}{\sqrt{\bar{p}}} \left(\frac{1}{2} \frac{\pi_{\phi}^2}{\bar{p}^3} - V(\phi) \right) \delta_a^i \delta E_j^a \delta_b^j \delta E_i^b. \quad (4)$$

where homogeneous part is given by

$$\bar{H}_m = V_0 \bar{N} \bar{p}^{3/2} \left(\frac{1}{2} \frac{\pi_{\phi}^2}{\bar{p}^3} + V(\phi) \right). \quad (5)$$

Here integration was constrained to fiducial volume V_0 . Further physical results do not depend of this quantity.

Energy density can be now defined as

$$\rho := \frac{1}{V_0 \bar{p}^{3/2}} \frac{\partial \bar{H}_m}{\partial \bar{N}}. \quad (6)$$

When matter content is defined one can derive particular form of the functions (2) and (3). Expressions for

the quantum holonomy corrections simplify to

$$T_Q = \frac{8\pi G}{3} \frac{\bar{p} \rho^2}{\rho_c}, \quad (7)$$

$$\Pi_{Qa}^i = \Pi_Q h_a^i = \frac{1}{2} \bar{p} \frac{\rho}{\rho_c} (2V - \rho) h_a^i. \quad (8)$$

These expressions were first derived in Ref. [18]. However we have found a discrepancy between the expression for Π_{Qa}^i derived here and this found in Ref. [18]. To approve the result presented here we show intermediate steps of derivation in Appendix. The difference is 1/3 factor inside the bracket. To derive these corrections we have applied so called $\bar{\mu}$ scheme of quantisation. Namely we used $\bar{\mu} = \sqrt{\Delta/\bar{p}}$ where $\Delta = 2\sqrt{3}\pi\gamma l_{Pl}^2$. It is well motivated to use this particular form of the function [20]. However, other choices are in principle also permitted. In this paper we consider only $\bar{\mu}$ scheme, which seems to be the best motivated.

Now equation for the tensor modes (1) simplifies to

$$\frac{d^2}{d\eta^2} h_a^i + 2\bar{k} \frac{d}{d\eta} h_a^i - \nabla^2 h_a^i + \tilde{T}_Q h_a^i = 0 \quad (9)$$

where we have defined the total holonomy correction

$$\tilde{T}_Q = T_Q - 16\pi G \Pi_Q = 16\pi G \bar{p} \frac{\rho}{\rho_c} \left(\frac{2}{3} \rho - V \right). \quad (10)$$

Therefore also source term correction has been included. This is in contrast with the analysis performed in [15, 16, 17], where this influence was neglected. In the classical theory in fact this term vanish in the linear order. Therefore when fluctuations of vacuum are considered, higher order term can be set to zero. However since, due to quantum corrections, source term contribute linearly, there is no reason to neglect this term. Therefore in the present paper we take it into account.

We introduce new common variable

$$u = \frac{ah_{\oplus}}{\sqrt{16\pi G}} = \frac{ah_{\otimes}}{\sqrt{16\pi G}}, \quad (11)$$

where $h_1^1 = -h_2^2 = h_{\oplus}$, $h_2^1 = h_1^2 = h_{\otimes}$ and $a = \sqrt{\bar{p}}$. Then performing the Fourier transform

$$u(\eta, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} u(\eta, \mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{x}}, \quad (12)$$

one can rewrite the equation (9) in the form

$$\frac{d^2}{d\eta^2} u(\eta, \mathbf{k}) + [k^2 + m_{\text{eff}}^2] u(\eta, \mathbf{k}) = 0, \quad (13)$$

where $k^2 = \mathbf{k} \cdot \mathbf{k}$ and

$$m_{\text{eff}}^2 = \tilde{T}_Q - \frac{a''}{a}. \quad (14)$$

In this paper we aim to solve equation (13). However first we must specify the background dynamics.

III. BACKGROUND DYNAMICS

Background dynamics is governed by the effective Friedmann equation

$$\left(\frac{1}{2\bar{p}}\frac{d\bar{p}}{dt}\right)^2 = \frac{\kappa}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right), \quad (15)$$

where

$$\rho_c = \frac{\sqrt{3}}{16\pi^2\gamma^3 l_{\text{Pl}}^4} \quad (16)$$

is critical energy density. This equation can be derived combining the Hamilton equation $\dot{\bar{p}} = \{\bar{p}, \bar{H}_m + \bar{H}_G^{\text{phen}}\}$ with the scalar constraint $\bar{H}_m + \bar{H}_G^{\text{phen}} = 0$.

Evolution of the scalar field component is governed by the Hamilton equations

$$\dot{\phi} = \{\phi, \bar{H}_m\} = \bar{p}^{-3/2}\pi_\phi, \quad (17)$$

$$\dot{\pi}_\phi = \{\pi_\phi, \bar{H}_m\} = -\bar{p}^{3/2}\frac{dV}{d\phi}. \quad (18)$$

Energy density and pressure of the homogeneous scalar field are expressed as follows

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad (19)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \quad (20)$$

A. Free scalar field

Energy density of the free scalar field has the form

$$\rho = \frac{1}{2}\frac{\pi_\phi^2}{\bar{p}^3} \quad (21)$$

and the solution of the effective Friedmann equation (effective background equation) is the following

$$\bar{p}(t) = (A + Bt^2)^{1/3} \quad (22)$$

where

$$A = \frac{1}{6}\kappa\pi_\phi^2\gamma^2\Delta, \quad B = \frac{3}{2}\kappa\pi_\phi^2. \quad (23)$$

Solution (22) represents non-singular bouncing evolution and is discussed in Ref. [9].

For further applications, it will be useful to relate the coordinate time with the conformal one $d\eta = dt/a(t)$. Assuming that $\eta(t=0) = 0$, we obtain

$$\eta(t) = \frac{t}{A^{1/6}} {}_2F_1\left[\frac{1}{2}, \frac{1}{6}, \frac{3}{2}; -\frac{B}{A}t^2\right]. \quad (24)$$

B. Scalar field with a multi-fluid potential

One can show [21] that the restriction $p_\phi = w\rho_\phi$, where $w = \text{const}$ in the framework of effective LQC, leads to the potential in the form

$$V(\phi) = \frac{1}{2}\rho_c(1-w)\frac{1}{\cosh^2\left[\sqrt{6\pi G(1+w)}\phi\right]}. \quad (25)$$

Solution of the equations of motion with this potential has simple analytic form

$$\bar{p}(t) = \bar{p}_c(1 + 6\pi G\rho_c(1+w)^2t^2)^{\frac{2}{3(1+w)}}. \quad (26)$$

It is worth to mention that for $w = 1$ and taking

$$\bar{p}_c^3 = A = \frac{1}{6}\kappa\pi_\phi^2\gamma^2\Delta \quad (27)$$

we recover the solution (22).

In analogy with the free field case we derive

$$\eta(t) = \frac{t}{\sqrt{\bar{p}_c}} {}_2F_1\left[\frac{1}{2}, \frac{1}{3(1+w)}, \frac{3}{2}; -6\pi G\rho_c(1+w)^2t^2\right]. \quad (28)$$

IV. ANALYTICAL CONSIDERATIONS

The theory of cosmological creation of particles bases on idea of “freezing” of the vacuum fluctuations. On the mathematical level this process can be seen as a squeezing and displacement of the vacuum state $|0\rangle$. This is equivalent with the creation of particles. For the non-interacting field theories the wave function is a product of the functions for the particular modes. Therefore the degree of squeezing and coherence can be different for the particular modes and is determined by the cosmological evolution. The typical scale for which squeezing and displacement of the vacuum becomes important is the Hubble scale. Modes of quantum fluctuations becomes classical (are described by the coherent states) when crossing the Hubble radius.

To describe process of particles creation quantitatively one can consider Bogolyubov transformation between initial and final states. Then computing the so-called Bogolyubov coefficients the number of produced particles can be obtained. However on the super-horizonal scales one can in principle obtain $\omega_k^2 = k^2 + m_{\text{eff}}^2 < 0$ and the interpretation in terms of particles fails. Then quantum state cannot be interpreted in terms of particles. Therefore, and from other reasons, it is useful to consider a correlation function which is well defined quantity for all energy scales. The correlation function for the tensor modes takes the form

$$\begin{aligned} \langle 0|\hat{h}_b^a(\mathbf{x}, \eta)\hat{h}_a^b(\mathbf{y}, \eta)|0\rangle &= 4\frac{16\pi G}{a^2} \int \frac{d^3\mathbf{k}}{(2\pi)^3} |u(k, \eta)|^2 e^{-i\mathbf{k}\cdot\mathbf{r}} \\ &= \int \frac{dk}{k} \mathcal{P}_T(k, \eta) \frac{\sin kr}{kr}, \end{aligned} \quad (29)$$

where we have defined the power spectrum

$$\mathcal{P}_T(k, \eta) = \frac{64\pi G}{a^2} \frac{k^3}{2\pi^2} |u(k, \eta)|^2. \quad (30)$$

The power spectrum can be related later to the amplitude of the CMB fluctuations. Therefore it is crucial to determinate this function.

Another way to describe physical properties of the quantum state is the mentioned method of Bogolyubov coefficients. The relation between annihilation and creation operators for the initial and for the final state is given by the Bogolyubov transformation

$$\hat{b}_{\mathbf{k}} = B_+(k)\hat{a}_{\mathbf{k}} + B_-(k)^* \hat{a}_{-\mathbf{k}}^\dagger, \quad (31)$$

$$\hat{b}_{\mathbf{k}}^\dagger = B_+(k)^* \hat{a}_{\mathbf{k}}^\dagger + B_-(k) \hat{a}_{-\mathbf{k}}, \quad (32)$$

where $|B_+|^2 - |B_-|^2 = 1$. Since we are working in the Heisenberg description the vacuum state does not change during the evolution. It results that $\hat{b}_{\mathbf{k}}|0_{\text{in}}\rangle = B_-(k)^* \hat{a}_{-\mathbf{k}}^\dagger|0_{\text{in}}\rangle$ is different from zero when $B_-(k)^*$ is a nonzero function. This means that in the final state the graviton field considered is no more in the vacuum state without particles. The number of produced particles in the final state is given by

$$n_{\mathbf{k}} = \frac{1}{2} \langle 0_{\text{in}} | [\hat{b}_{\mathbf{k}}^\dagger \hat{b}_{\mathbf{k}} + \hat{b}_{-\mathbf{k}}^\dagger \hat{b}_{-\mathbf{k}}] | 0_{\text{in}} \rangle = |B_-(k)|^2. \quad (33)$$

The energy density of gravitons is given by

$$d\rho_{\text{gw}} = 2 \cdot \hbar\omega \cdot \frac{4\pi\omega^2 d\omega}{(2\pi c)^3} \cdot |B_-(k)|^2. \quad (34)$$

where we used definition (33). To describe the spectrum of gravitons it is common to use the parameter

$$\Omega_{\text{gw}}(\nu) = \frac{\nu}{\rho_*} \frac{d\rho_{\text{gw}}}{d\nu} \quad (35)$$

where ρ_{gw} is the energy density of gravitational waves and ρ_* is the present critical energy density.

A. Free scalar field

Based on solution (22) we derive

$$m_{\text{eff}}^2 = \frac{\kappa^2 \pi_\phi^4}{4} \frac{(t^2 + \frac{1}{9}\gamma^2 \Delta)}{(A + Bt^2)^{5/3}} \geq 0. \quad (36)$$

We show this function in Fig. 1. We compare it with the classical expression

$$m_{\text{eff}}^2(\tilde{T}_Q = 0) = \frac{\kappa^2 \pi_\phi^4}{4} \frac{(t^2 - \frac{1}{3}\gamma^2 \Delta)}{(A + Bt^2)^{5/3}}. \quad (37)$$

The difference is significant since now effective mass is a non-negative function, $m_{\text{eff}}^2 \geq 0$. One can also compare this with the case when the source term corrections

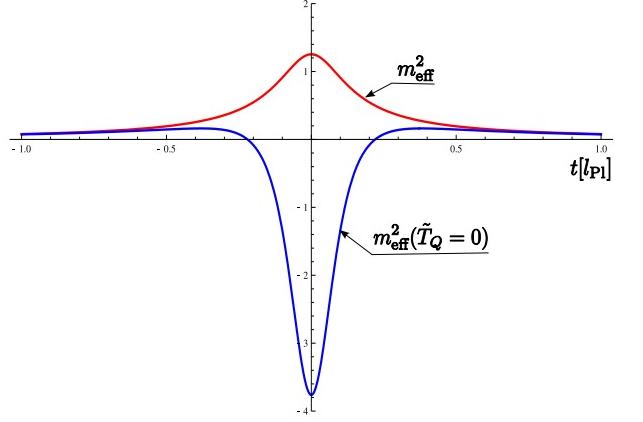


FIG. 1: Evolution of the effective masses m_{eff}^2 and $m_{\text{eff}}^2(\tilde{T}_Q = 0)$. In this figure we have assumed $\pi_\phi = 0.1 l_{\text{Pl}}$

were neglected. Then, as can be found in Ref. [16], the effective mass is negative in some regime and behaves like the classical one. Here difference is crucial and has important consequences. Namely since $m_{\text{eff}}^2 \geq 0$ we have $\omega_k^2 \geq 0$ and interpretation in terms of particles is possible on all scales. This becomes a nice property also when the Hamiltonian of the perturbations is minimized to find a proper vacuum state. It can be shown that when $\omega_k^2 \geq 0$, the Hamiltonian has a minimum for all k and a well defined vacuum can be found. Otherwise for some $k < k_x$, the lowest-energy instantaneous vacuum state does not exist.

Now we are going to consider the pre-bounce limit. Taking $|t| \rightarrow \infty$, we find

$$m_{\text{eff}}^2 \rightarrow \frac{1}{4} \frac{1}{\eta^2}. \quad (38)$$

The normalised solution of the equation (13) has the form

$$u(k, \eta) = \sqrt{\frac{\pi}{2}} e^{i\pi/4} \frac{1}{\sqrt{2k}} \sqrt{-\eta k} H_0^{(1)}(-\eta k). \quad (39)$$

We have chosen here advanced modes and performed normalisation with use of the Wronskian condition. In the super-horizontal limit $-\eta k \ll 1$ we can apply the approximation

$$H_0^{(1)}(x) \simeq 1 + i \frac{2}{\pi} \left[\ln \left(\frac{x}{2} \right) + \gamma_E \right], \quad (40)$$

where $\gamma_E = 0.57721\dots$ is Euler-Mascheroni constant. Expression for the power spectrum in the superhorizontal limit is therefore

$$\mathcal{P}_T(k) = \mathcal{A} k^3 \left\{ 1 + \frac{4}{\pi^2} \left[\ln \left(-\frac{k\eta}{2} \right) + \gamma_E \right]^2 \right\}, \quad (41)$$

where

$$\mathcal{A} = 4\sqrt{\frac{2}{\pi}} \left(\frac{3}{2} \right)^{1/6} \left(\frac{l_{\text{Pl}}}{\pi_\phi} \right). \quad (42)$$

To investigate k dependence in formula (41) we define the spectral index

$$n_T = \frac{d \ln \mathcal{P}_T(k)}{d \ln k} \quad (43)$$

and obtain

$$n_T = 3 + \frac{8}{\pi^2} \frac{\ln\left(-\frac{k\eta}{2}\right) + \gamma_E}{1 + \frac{4}{\pi^2} \left[\ln\left(-\frac{k\eta}{2}\right) + \gamma_E\right]^2}. \quad (44)$$

We show this function for some fixed time in Fig. 2. We find that the resulting spectral index is blue and ap-

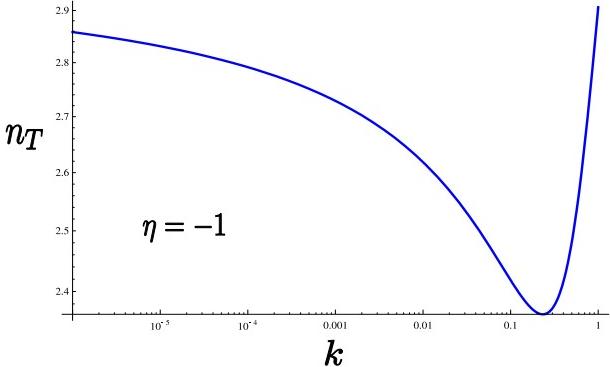


FIG. 2: Running spectral index on the super-horizontal scales.

proaching $n_T = 3$ for $k \rightarrow 0$. This blue-tilted spectrum was predicted earlier in [16]. Recent investigations suggest that also for inflationary cosmology with holonomy corrections obtained spectrum is blue-tilted and $n_T = 3$ at super-horizontal scales [17].

B. Multi-fluid potential

Now we are going to perform similar analysis for the model with a multi-fluid potential. We obtain formula

$$m_{\text{eff}}^2 = \bar{p}_c \frac{1}{8} \kappa^2 \rho_c^2 (1+w)^2 (3w-1) (1+6\pi G \rho_c (1+w)^2 t^2)^\alpha \times \left\{ t^2 - \frac{4}{3} \frac{\Delta \gamma^2}{(1+w)(3w-1)} \left[1 - \frac{2}{3} \frac{(1+3w)}{(1+w)} \right] \right\}, \quad (45)$$

where

$$\alpha = -\frac{2}{3} \frac{2+3w}{1+w}. \quad (46)$$

In the limit $|t| \rightarrow \infty$ we obtain

$$m_{\text{eff}}^2 \rightarrow \frac{6w-2}{(1+3w)^2} \frac{1}{\eta^2}, \quad (47)$$

where we changed time to conformal. Advanced and normalised solution of the equation (13) in the considered limit is

$$u(k, \eta) = \sqrt{-k\eta} \sqrt{\frac{\pi}{4k}} e^{i\frac{\pi}{2}(|\nu|+\frac{1}{2})} H_{|\nu|}^{(1)}(-\eta k) \quad (48)$$

where

$$\nu^2 = \frac{9}{4} \frac{(1-w)^2}{(1+3w)^2}. \quad (49)$$

Power spectrum of the perturbations is then given as

$$\mathcal{P}_T(k) \propto (-k\eta)^{3-2|\nu|}, \quad (50)$$

where super-horizontal approximation

$$H_n^{(1)}(x) \simeq -\frac{i}{\pi} \Gamma(n) \left(\frac{x}{2}\right)^{-n} \text{ for } x \ll 1 \quad (51)$$

has been used. It must be stressed that the above formula does not hold for $n = 0$ ($w=1$). In that case another expansion (40) must be applied.

It is worth to mention that scale invariant spectrum $|\nu| = \frac{3}{2}$ is recovered both for $w = -1$ and $w = 0$, as it can be directly seen from (49). This duality was investigated in Ref. [22] in context of the free scalar field perturbations.

C. Sub-horizontal solutions

Since now we were only concerned with the pre-bounce phase. Now we are going to evolve modes through the bounce. We firstly consider the case of modes which stay under the Hubble radius before the bounce. For that modes the initial vacuum state is given by

$$u_{\text{in}} = \frac{e^{-ik\eta}}{\sqrt{2k}}. \quad (52)$$

This can be obtained as a limit of the mode function (39) for $-k\eta \gg 1$. To be specific, let us consider the model with a free scalar field and $\pi_\phi = 0.1 l_{\text{Pl}}$.

At the Hubble radius we have

$$k_{\text{H}} = a|H| \quad (53)$$

which is shown in Fig. 3. We see that for initial time, let us say $t = -1000l_{\text{Pl}}$, all modes with $k > 0.003$ are well described by the function (52). This solutions however do not hold during the phase of bounce. Close to the bounce one can approximate

$$m_{\text{eff}}^2 \approx m_{\text{eff}}^2(t=0) = \frac{1}{(54)^{1/3}} \kappa (\pi_\phi \rho_c)^{2/3} \equiv k_0^2. \quad (54)$$

For the considered conditions we obtain $k_0 \simeq 1.12$. In this approximation the solutions during the bounce phase are

$$u_{\text{bounce}} = \frac{A_k}{\sqrt{2\Omega}} e^{-i\Omega\eta} + \frac{B_k}{\sqrt{2\Omega}} e^{i\Omega\eta}, \quad (55)$$

where $\Omega = \sqrt{k^2 + k_0^2}$. Finally, in the post-bounce phase we have a superposition of advanced and retarded modes

$$u_{\text{out}} = \frac{\alpha_k}{\sqrt{2k}} e^{-ik\eta} + \frac{\beta_k}{\sqrt{2k}} e^{ik\eta}. \quad (56)$$

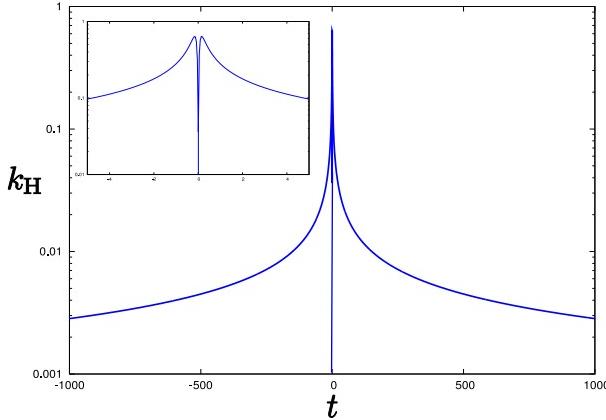


FIG. 3: Evolution of the Hubble wave number $k_H = a|H|$. Here $\pi_\phi = 0.1l_{\text{Pl}}$.

Here the relation $|\alpha_k|^2 - |\beta_k|^2 = 1$ holds, as a consequence of the normalisation condition. Now we have to match solutions from the three considered regions to determinate coefficients α_k and β_k . In order to do that we must specify a time when the matching is performed. We choose it in the mirror points $-t_- = t_+$ where H^2 reaches its maximal value. Then $-t_- = t_+ = t_0$ where

$$t_0 = \frac{1}{\sqrt{24\pi G\rho_c}}. \quad (57)$$

With use of equation (24) we obtain

$$\eta_0 = \eta(t_0) = \frac{{}_2F_1\left[\frac{1}{2}, \frac{1}{6}, \frac{3}{2}; -1\right]}{\sqrt{3}\kappa\rho_c^{1/3} \left(\pi_\phi^2/2\right)^{1/6}}. \quad (58)$$

For the considered setup we obtain $\eta_0 \simeq 0.285$.

In order to derive formulas for the coefficients α_k and β_k , we define the matrices

$$\mathbf{M}_0 = \begin{pmatrix} \frac{e^{-ik\eta_-}}{\sqrt{2k}} & \frac{e^{ik\eta_-}}{\sqrt{2k}} \\ -i\sqrt{\frac{k}{2}}e^{-ik\eta_-} & i\sqrt{\frac{k}{2}}e^{ik\eta_-} \end{pmatrix}, \quad (59)$$

$$\mathbf{M}_1 = \begin{pmatrix} \frac{e^{-i\Omega\eta_-}}{\sqrt{2\Omega}} & \frac{e^{i\Omega\eta_-}}{\sqrt{2\Omega}} \\ -i\sqrt{\frac{\Omega}{2}}e^{-i\Omega\eta_-} & i\sqrt{\frac{\Omega}{2}}e^{i\Omega\eta_-} \end{pmatrix}, \quad (60)$$

$$\mathbf{M}_2 = \begin{pmatrix} \frac{e^{-i\Omega\eta_+}}{\sqrt{2\Omega}} & \frac{e^{i\Omega\eta_+}}{\sqrt{2\Omega}} \\ -i\sqrt{\frac{\Omega}{2}}e^{-i\Omega\eta_+} & i\sqrt{\frac{\Omega}{2}}e^{i\Omega\eta_+} \end{pmatrix}, \quad (61)$$

$$\mathbf{M}_3 = \begin{pmatrix} \frac{e^{-ik\eta_+}}{\sqrt{2k}} & \frac{e^{ik\eta_+}}{\sqrt{2k}} \\ -i\sqrt{\frac{k}{2}}e^{-ik\eta_+} & i\sqrt{\frac{k}{2}}e^{ik\eta_+} \end{pmatrix}. \quad (62)$$

Then matching conditions can be economically written as

$$\begin{pmatrix} \alpha_k \\ \beta_k \end{pmatrix} = \mathbf{M}_3^{-1} \mathbf{M}_2 \mathbf{M}_1^{-1} \mathbf{M}_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad (63)$$

Multiplying these matrices we obtain

$$\alpha_k = \frac{-i \cos(2\eta_0 k) + \sin(2\eta_0 k)}{2k\Omega} \times \\ \times [2ik\Omega \cos(2\eta_0 \Omega) + (k^2 + \Omega^2) \sin(2\eta_0 \Omega)] \quad (64)$$

$$\beta_k = -\frac{i(k^2 - \Omega^2) \sin(2\eta_0 \Omega)}{2k\Omega} \quad (65)$$

The resulting square of the amplitude for the out state modes is

$$|u_{\text{out}}|^2 = \frac{(k^2 + \Omega^2)^2 - k_0^4 \cos[4\eta_0 \Omega] - k_0^2 \sin[2\eta_0 \Omega] ((k + \Omega)^2 \sin[2k(\eta - \eta_0) + 2\Omega\eta_0] - (k - \Omega)^2 \sin[2k(\eta - \eta_0) - 2\Omega\eta_0])}{8k^3(k^2 + k_0^2)}. \quad (66)$$

Based on this result one can calculate power spectrum of perturbations. We show this spectrum in Fig. 4. Obtained spectrum exhibits sub-horizontal oscillations. This effect can be intuitively understood when analogy with Schrödinger equation is employed. Namely the mode equations are equivalent to a one dimensional Schrödinger equation with potential $V = -m_{\text{eff}}^2$. Here spatial variable is replaced by the conformal time η . In the employed approximation potential is square well of width $2\eta_0$ and depth $m_{\text{eff}}^2(t=0)$. Therefore the evolution of the given mode can be seen as transition of a particle over the potential well. Amplifications of the amplitude of transmission correspond to resonances between

the width of the potential and the phase shift.

It can be shown that the obtained coefficients (α_k, β_k) are in fact the Bogolyubov coefficients $\alpha_k = B_+$ and $\beta_k = B_-$. Therefore the number of produced gravitons is given by

$$n_k = |\beta_k|^2 = \frac{k_0^4 \sin^2(2\eta_0 \sqrt{k^2 + k_0^2})}{k^2(k_0^2 + k^2)}. \quad (67)$$

We show this dependence in Fig. 5. Now it is straightforward to calculate the parameter Ω_{gw} . We show this function in Fig. 6. We compare it with the obtained low energy approximation $\Omega_{\text{gw}} \propto \nu^{-2}$. Obtained values

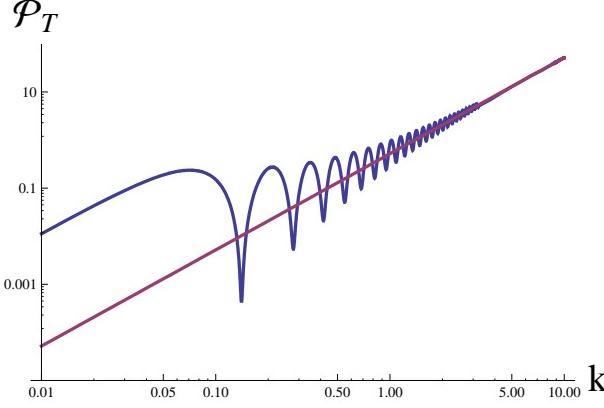


FIG. 4: Oscillating tensor power spectrum of the subhorizontal modes at $t = 50 l_{\text{Pl}}$. Straight line represents spectrum of reference $\mathcal{P}_T \propto k^2$.

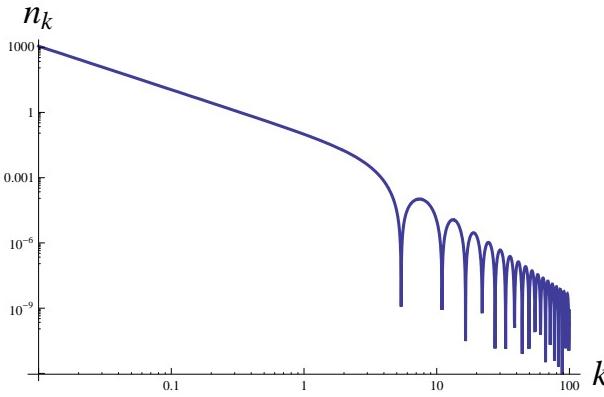


FIG. 5: Occupation number of the gravitons in the postbounce state.

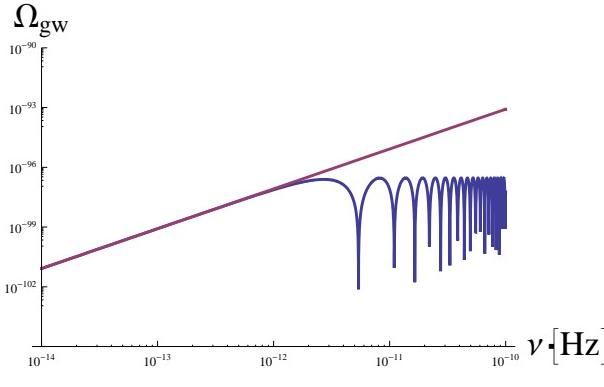


FIG. 6: Parameter Ω_{gw} (bottom curve). Straight line represents low energy approximation $\Omega_{\text{gw}} \propto \nu^{-2}$.

of Ω_{gw} are many orders of magnitude below the present threshold for detection. The results obtained were performed for some simplified model and for the fixed value of π_ϕ . In particular in the low energy limit $\Omega_{\text{gw}} \sim \pi_\phi^{2/3}$, therefore effect of varying π_ϕ is considerable. However, we do not expect significant changes due to the approxi-

mations performed. This statement will be confirmed by the numerical simulations in Section V.

D. Super-horizontal solutions

In the previous subsection we have shown solutions of the mode equation (13) in the sub-horizontal limit. Now we are going to study the super-horizontal $k \rightarrow 0$ limit.

We introduce new variable in the form

$$f = \sqrt{a}u, \quad (68)$$

and change the conformal time to the coordinate one $dt = ad\eta$. Then equation (13) can be rewritten in the form

$$\frac{d^2 f}{dt^2} + \Omega^2(k, t)f = 0. \quad (69)$$

Here the parameter $\Omega^2(k, t)$ is defined as follows

$$\Omega^2(k, t) = \left(\frac{k}{a}\right)^2 + \epsilon \frac{\tilde{T}_Q}{a^2} - \frac{3}{2} \left(\frac{\ddot{a}}{a} + \frac{1}{2}H^2\right). \quad (70)$$

We have introduced here parameter the ϵ to trace effects of the holonomy corrections in the later equations. In the classical limit we should take $\epsilon = 0$ while in presence of the holonomy corrections $\epsilon = 1$. Taking $k = 0$ and introducing new complex variable $z \in \mathbb{C}$,

$$z = \frac{1}{2} + i \frac{1}{2} \sqrt{6\pi G\rho_c} (1+w)t \quad (71)$$

we can rewrite equation (69) in the following form

$$\frac{d^2 f}{dz^2} + Q(z)f = 0 \quad (72)$$

where

$$Q(z) = \frac{\alpha_2 z^2 + \alpha_1 z + \alpha_0}{z^2(z-1)^2}. \quad (73)$$

The coefficients are

$$\alpha_0 = \frac{9(1+2w) - 4\epsilon(1+3w)}{36(1+w)^2}, \quad (74)$$

$$\alpha_1 = -\frac{w}{(1+w)^2}, \quad (75)$$

$$\alpha_2 = \frac{w}{(1+w)^2} \quad (76)$$

and it will be useful later to remember that $\alpha_1 + \alpha_2 = 0$. Now introducing the new variable

$$f(z) = z^L (z-1)^K g(z) \quad (77)$$

with

$$L = \frac{c}{2} \quad (78)$$

$$K = \frac{a+b+1-c}{2} \quad (79)$$

one can rewrite equation (72) as a hypergeometric equation

$$z(1-z)\frac{d^2g}{dz^2} + [c - (a+b+1)z]\frac{dg}{dz} - abg = 0. \quad (80)$$

Solution of this equation is given by the hypergeometric functions

$$g(z) = C_2 F_1(a, b, c; z). \quad (81)$$

Furthermore, we have a system of equations for the coefficients

$$\alpha_0 + L(L-1) = 0, \quad (82)$$

$$\alpha_1 + ab - 2KL - 2L(L-1) = 0, \quad (83)$$

$$\alpha_2 - ab + 2KL + L(L-1) + K(K-1) = 0. \quad (84)$$

One can find that, since $\alpha_1 + \alpha_2 = 0$, we have either $K = L$ or $K = 1 - L$, where $L = \frac{1}{2}(1 \pm \sqrt{1 - 4\alpha_0})$. For $K = L$ we find

$$a = \frac{1}{2}(2c - 1 \pm \sqrt{1 + 4\alpha_1}), \quad (85)$$

$$b = 2c - a - 1, \quad (86)$$

$$c = 2L = 1 \pm \sqrt{1 - 4\alpha_0}, \quad (87)$$

and while $K = 1 - L$ we have

$$a = \frac{1}{2}(1 \pm \sqrt{1 + 4\alpha_1}), \quad (88)$$

$$b = 1 - a, \quad (89)$$

$$c = 2L = 1 \pm \sqrt{1 - 4\alpha_0}. \quad (90)$$

As an exemplary solution we consider $w = 1$ case both classically ($\epsilon = 0$) and with holonomy corrections to the mode equation ($\epsilon = 1$). Then since $\alpha_1 = -\alpha_2 = -1/4$ we have

$$a = b = c - \frac{1}{2} \quad (91)$$

where

$$c_{\pm}(\epsilon = 1) = 1 \pm \frac{\sqrt{47}}{6} \text{ and } c_{\pm}(\epsilon = 0) = 1 \pm \frac{\sqrt{7}}{2} \quad (92)$$

In Fig. 7 we show solutions for the real components of h variable.

In Fig. 8 we show solutions for the imaginary components of the h variable.

In Fig. 9 we show solutions for the absolute value of the h variable.

As it can be seen, solutions with and without the quantum holonomy corrections are qualitatively similar. Another observations is that for the times $t \gg 1$ evolution takes a logarithmic form. This result can be understood considering equation (13) in the classical limit and taking $k \rightarrow 0$. Then one can find approximate solution in the form

$$h \simeq A_k + B_k \int^{\eta} \frac{d\eta'}{a^2(\eta')} \quad (93)$$

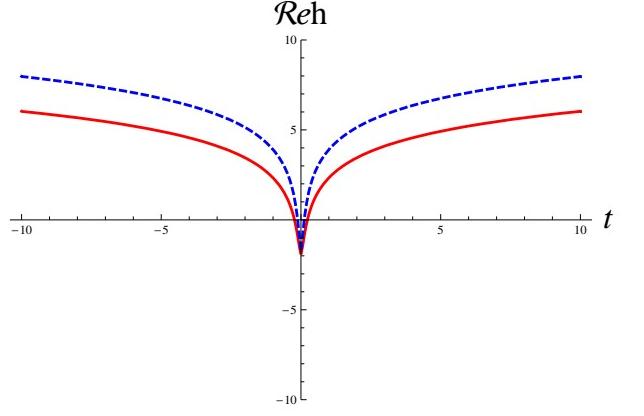


FIG. 7: Real components of h variable. Dashed (blue) line represents solution with $c_+(\epsilon = 1)$ while straight (red) line represents solution with $c_+(\epsilon = 0)$.

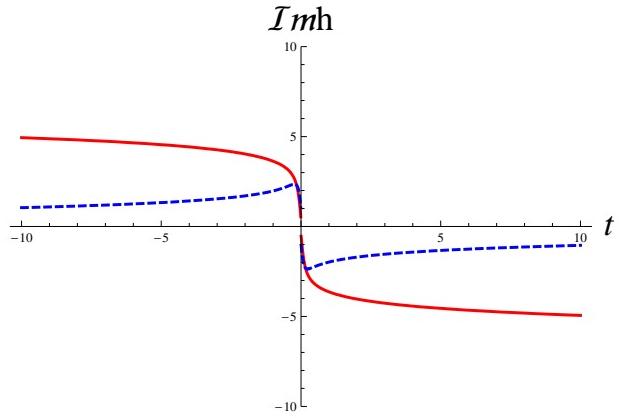


FIG. 8: Imaginary components of h variable. Dashed (blue) line represents solution with $c_+(\epsilon = 1)$ while straight (red) line represents solution with $c_+(\epsilon = 0)$.

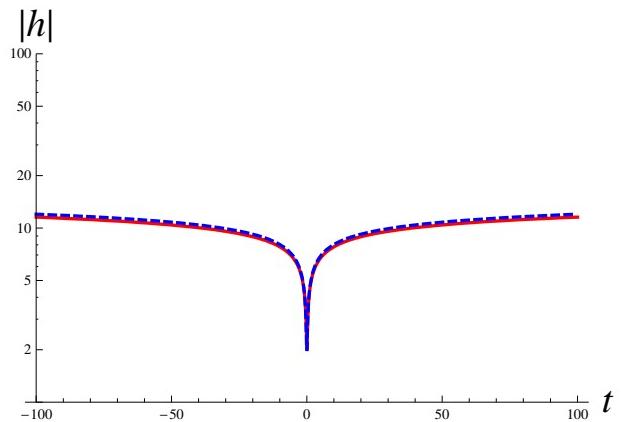


FIG. 9: Absolute values of h variable. Dashed (blue) line represents solution with $c_+(\epsilon = 1)$ while straight (red) line represents solution with $c_+(\epsilon = 0)$.

where A_k and B_k are some constants. For the models considered in the present paper we have $a \propto (\pm\eta)^{2/(1+3w)}$

and $\pm\eta \propto (\pm t)^{\frac{1+3w}{3(1+w)}}$. Here we have + sign for the expanding phase and - sign for the contracting one. Therefore for the considered $w = 1$ case we find

$$h \simeq \tilde{A}_k + \tilde{B}_k \ln(\pm t) \text{ for } |t| \gg 1 \quad (94)$$

in agreement with the solutions found in this subsection.

V. NUMERICAL INVESTIGATIONS

As it was shown in the previous section, analytic solutions of the mode equation are available only in some limits. Namely for both $t^2 \rightarrow \infty$ and $k \rightarrow 0$. Also for $k \rightarrow \infty$ an approximate solution was found. It is however not sufficient to describe whole spectrum of the gravitational waves produced on the bounce phase since the interesting intermediate regimes are unexplored. Therefore numerical analysis is required.

In the numerical computations we are going to solve the autonomous system of equations

$$\frac{du}{d\eta} = \pi_u, \quad (95)$$

$$\frac{d\pi_u}{d\eta} = -[k^2 + m_{\text{eff}}^2(t)] u, \quad (96)$$

$$\frac{dt}{d\eta} = a(t), \quad (97)$$

where $a(t)$ and $m_{\text{eff}}^2(t)$ are defined for particular background dynamics. In the considered models with free scalar field and multi-fluid potential these functions are given by analytical expressions. Since canonical variables $u, \pi_u \in \mathbb{C}$ we decompose

$$u = u_1 + iu_2, \quad (98)$$

$$\pi_u = \pi_{u1} + i\pi_{u2}. \quad (99)$$

Now it is crucial to define proper initial conditions for $(u_1, u_2, \pi_{u1}, \pi_{u2})$ for some time η_0 . It is always unambiguous how to choose a proper vacuum defined on the cosmological background. However on the sub-horizontal scales, when Minkowski space approximation holds, we can set

$$u_1(\eta_0) = \frac{1}{\sqrt{2k}} \cos(k\eta_0), \quad (100)$$

$$u_2(\eta_0) = -\frac{1}{\sqrt{2k}} \sin(k\eta_0) \quad (101)$$

and

$$\pi_{u1}(\eta_0) = -\sqrt{\frac{k}{2}} \sin(k\eta_0), \quad (102)$$

$$\pi_{u2}(\eta_0) = -\sqrt{\frac{k}{2}} \cos(k\eta_0) \quad (103)$$

at some time η_0 . Here we set initial values like in the model of sub-horizontal modes studied in the previous

section. Therefore analysis is correct for the modes with $k > 0.003$.

In Fig. 10 we plot evolution of the $k = 0.1$ mode during the bounce phase. We compare here the evolu-

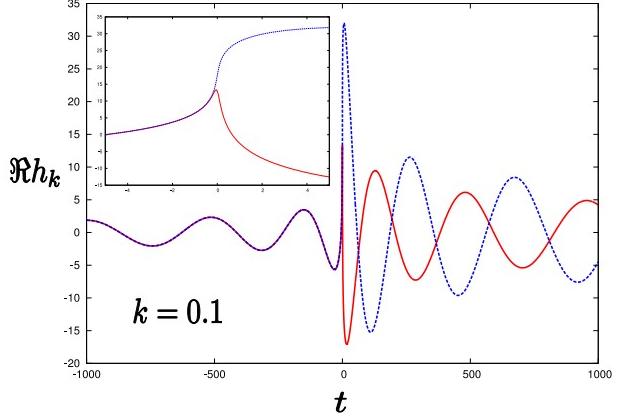


FIG. 10: Evolution of the modes with $k = 0.1$. Dotted (blue) curve represent solution of mode equations with holonomy corrections. Straight (red) curve represent solution of mode equations without holonomy effects.

tion of modes with and without holonomy corrections to the modes equation. We see that close to the turning point the effects of the holonomy corrections become significant. However the further oscillating evolution does not change qualitatively. The difference is some suppression of the amplitude of perturbations due to the quantum corrections. This feature can be also seen in Fig. 11 where classical and quantum corrected tensor power spectra are shown. We find, comparing with the clas-

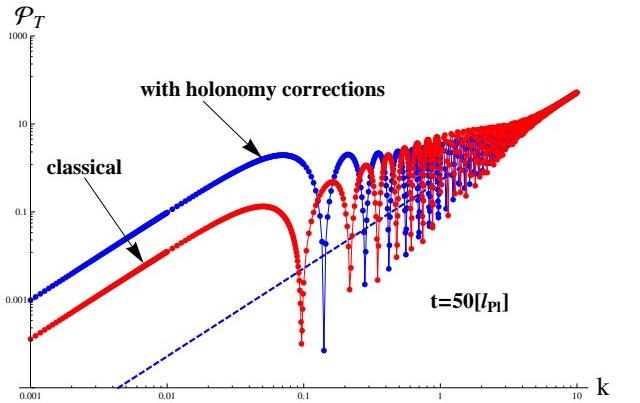


FIG. 11: Comparison between sub-horizontal spectra obtained with and without holonomy corrections to mode equation. Dashed (blue) line represents rescaled initial vacuum power spectrum.

sical case, that quantum holonomy effects amplify low energy modes. Therefore tensor power spectra increases by about one order of magnitude. For the high energies classical spectra starts to dominate slightly. It is also worth to notice that oscillations do not overlap.

To impose initial conditions on the super-horizontal scales one can use instantaneous vacuum. This is however possible only for values of k fulfilling $\omega_k^2 \geq 0$. As we have found earlier this condition is fulfilled for all k in the model with the free scalar field. Therefore initial instantaneous vacuum state can be defined on all length scales. It can be shown that Hamiltonian of perturbations at time η_0 is minimised for

$$u(\eta_0) = \frac{1}{\sqrt{2\omega_k}} \quad (104)$$

$$\pi_u(\eta_0) = -i\sqrt{\frac{\omega_k}{2}} \quad (105)$$

where $\omega_k = \sqrt{k^2 + m_{\text{eff}}^2}$.

In Fig. 12 we show tensor power spectrum at post-bounce stage ($t = 50 l_{\text{Pl}}$) with imposed instantaneous vacuum initial conditions at $t = -1000 l_{\text{Pl}}$. The charac-

part becomes nearly flat (depending on the model of inflation). It is possible that the oscillating features will also survive giving the footprints of the bouncing phase. However the further analysis has to be performed to approve these speculations. In particular inflationary power spectrum with the obtained bouncing initial conditions must be calculated. Then it will be possible to compute the B-type polarization spectra of CMB. Therefore a way to relate the quantum cosmological effects with the low energy physics becomes potentially available.

VI. SUMMARY

In this paper we have considered influence of Loop Quantum Gravity effects on the gravitational waves propagation in the flat FRW cosmological background. The considerations presented based on the semi-classical approach where quantum effects are introduced by corrections to the classical equations of motion. This approach was successfully applied to the homogeneous models. In this case good agreement between results of fully quantum and semi-classical analyses was found. Here we have applied semi-classical approach to the inhomogeneous model where inhomogeneity is treated perturbatively. Therefore perturbations had no influence on background. In general both classical and quantum back-reaction effects can be important close to the phase of bounce. Here we assumed that they can be neglected. We have also not considered effects of the quantum fluctuations of background on the inhomogeneities. Quantum effects were introduced by the so called holonomy corrections. In the homogeneous models these corrections lead to absence of the initial singularity and emergence of the bounce phase. Effects of the other known type of LQG corrections, the inverse volume ones, were studied earlier in numerous papers. Here we considered self-consistent model where holonomy effects influence both background and perturbations (gravitational waves). In the earlier studies effects on background and perturbations were studied independently. In particular in Ref. [16] a model of gravitational waves production during the holonomy-induced bounce phase was investigated. In Ref. [15, 17] effects of holonomy corrections on the gravitational waves in inflationary phase were studied. However quantum effects on the background dynamics were neglected there. Moreover quantum-corrected source term was not taken into account in those studies. Linear part of this term vanish in the classical limit. However, its contribute while holonomy corrections are present. Therefore source term has to be taken into account in the full treatment. In the present paper we have included effects of this term.

We have considered models with both free scalar field and self-interacting field with multi-fluid potential. In both cases scalar field is a monotonic function and can be treated as a internal time variable.

We have shown that in the model with the free field, effective mass term m_{eff}^2 for gravitational waves is a non-

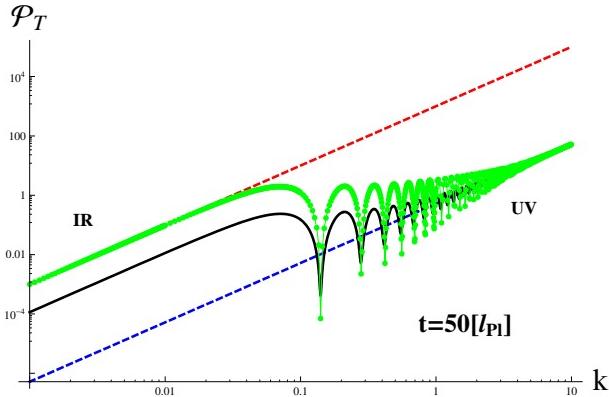


FIG. 12: Whole post-bounce tensor spectra in presence of the holonomy corrections. Green points comes from the numerical simulations. Black line is the analytical spectrum from the model given by Eq. 66. Dashed (red and blue) lines represents UV and IR behaviours, in both cases $P_T \propto k^2$.

teristic feature of the spectrum are oscillations. Moreover both UV and IR behaviours are in the form $P_T \propto k^2$. We see that analytical model given by Eq. 66 fairly good overlap with the numerical results. Especially structure of oscillations is exactly recovered. Also the asymptotic behaviours are consistent. The evident discrepancy is the difference in the total amplitude. In fact this difference can be suitably adjusted varying parameters of the model η_0 and k_0 . Then low energy behaviour can be exactly recovered. However it introduces additional phase shift and structures of oscillation no longer overlap. It is also important to note that effect of the imposed instantaneous vacuum initial conditions is negligible in the range studied. Therefore Minkowski vacuum approximation is still valid.

The obtained power spectrum can be now applied as an initial condition for the inflationary modes. We expect that super-horizontal part of the spectrum does not change during the inflationary phase. However the UV

negative function. This is not the case for the models with multi-fluid potential. We have found solutions of the mode function in the pre-bounce phase and determined the power spectra of the obtained perturbations. Then we have considered sub-horizontal solutions during the bounce phase. We matched the solutions from pre-bounce, bounce and post-bounce phases. Based on this we have found power spectrum of gravitational waves and determined Bogolyubov coefficients. Then number of produced gravitons n_k and the parameter Ω_{gw} were calculated. We have found that Ω_{gw} reaches 10^{-96} which is far below any observational bounds. These results were obtained for fixed parameter $\pi_\phi = 0.1 l_{\text{Pl}}$.

Based on analytical considerations we have found that power spectrum exhibits oscillations on sub-horizontal scales. An intuitive explanation of this effect was given. We have also solved the model analytically in the super-horizontal limit. These results indicate that quantum corrections do not introduce qualitative difference in the power spectrum on these scales. Therefore the obtained lack of power on the large scales is a feature of the bouncing evolution and not of the quantum corrections to the mode equation.

Subsequently we have investigated the model numerically. We have approved presence of the oscillations emerged from the simplified analytical considerations. Both numerical and analytical results were compared. We have found good qualitative and quantitative agreement. We have also approved earlier observation that quantum corrections does not introduce qualitative difference in the power spectrum. The only differences observed were in total amplitude and phase of oscillations.

Imposing initial instantaneous vacuum state we have also studied the low energy part of the power spectrum. Therefore we have finally found the full shape of the tensor power spectrum. This spectrum can be used to study further phenomenological consequences. In particular, it can be applied as an initial condition for the inflationary modes. Then we expect that the sub-horizontal part of the spectrum becomes flat while super-horizontal form survive. It is also possible that sub-horizontal oscillations survive as features of the dominant nearly flat inflationary spectrum. Therefore two observational effects of the bouncing phase can be distinguished: oscillations and lack of power on the super-horizontal scales. These effects can potentially be tested with the future CMB missions like Planck [23] or proposed CMBPol [24]. Especially promising are observations of the CMB polarization. Here bounce can lead to the low multipoles suppression in the B-type spectrum. At present projects like Clover [25], QUAID [26] or QUIET [27] are aiming to detect this spectrum and first results are expected in the near future. Therefore it is the next step to derive quantitative predictions of the CMB features from the presented model.

Acknowledgments

Author is grateful to Francesca Vidotto for discussion during the conference "Quantum Gravity in Cracow²" 19-21 XII 2008, Poland, where part of these results were presented.

APPENDIX A: LOOP QUANTUM GRAVITY WITH GRAVITATIONAL WAVES

Loop Quantum Gravity (LQG) describes the gravitational field as $SU(2)$ non-Abelian gauge field using background independent methods. The canonical fields are so called Ashtekar variables ($A = A_a^i \tau_i dx^a$, $E = E_i^a \tau^i \partial_a$) which take value in $\mathfrak{su}(2)$ and $\mathfrak{su}(2)^*$ algebras respectively and they fulfil the Poisson bracket

$$\{A_a^i(\mathbf{x}), E_j^b(\mathbf{y})\} = \gamma \kappa \delta_a^b \delta^i_j \delta^{(3)}(\mathbf{x} - \mathbf{y}) \quad (\text{A1})$$

where $\kappa = 8\pi G$ and γ is the Barbero-Immirzi parameter. These variables are analogues of the vector potential and the electric field in electrodynamics. The Ashtekar variables are related with triad representation. In LQG gauge fields describe only spatial part Σ when time is treated separately.

In cosmological applications we perturb basic variables around a background

$$E_i^a = \bar{E}_i^a + \delta E_i^a, \quad (\text{A2})$$

$$A_a^i = \bar{A}_a^i + \delta A_a^i. \quad (\text{A3})$$

For the spatially flat FRW background components have the following form

$$\bar{E}_i^a = \bar{p} \delta_i^a, \quad (\text{A4})$$

$$\bar{A}_a^i = \gamma \bar{k} \delta_a^i, \quad (\text{A5})$$

where $\bar{p} = a^2$ and $\bar{k} = \dot{p}/2\bar{p}$. Perturbations can be split for the scalar, vector and tensor parts. For the purpose of this paper we consider here only the gravitational waves (tensor part). Tensor perturbations of the flat FRW metric are introduced as follows

$$\begin{aligned} g_{00} &= -N^2 + q_{ab} N^a N^b = -\bar{N} = -a^2, \\ g_{0a} &= q_{ab} N^b = 0, \\ g_{ab} &= q_{ab} = a^2 [\delta_{ab} + h_{ab}], \end{aligned}$$

with the conditions $h_a^a = \partial_a h_b^a = 0$ and $|h_{ab}| \ll 1$. In the TT gauge $h_1^1 = -h_2^2 = h_\oplus$ and $h_2^1 = h_1^2 = h_\otimes$.

Now we are going to perturb the Hamiltonian of the theory. The full Hamiltonian is composed of the gravitational and H_G and matter H_m parts. Hamiltonian H_G takes the form of a liner combination of the constraints

$$H_G = \int_{\Sigma} d^3x (N^i G_i + N^a C_a + NS).$$

Spatial diffeomorphisms constraint:

$$C_a = E_i^b F_{ab}^i - (1 - \gamma^2) K_a^i G_i.$$

Gauss constraint:

$$G_i = D_a E_i^a = \partial_a E_i^a + \epsilon_{ijk} A_a^j E_k^a.$$

Scalar constraint:

$$S = \frac{E_i^a E_j^b}{\sqrt{\det E}} \left[\varepsilon^{ij}_k F_{ab}^k - 2(1 + \gamma^2) K_{[a}^i K_{b]}^j \right]$$

where $F = dA + \frac{1}{2}[A, A]$. However thanks to the quantum gravity effect this Hamiltonian undergoes modifications. These modifications can be introduced on the phenomenological level by the replacement

$$\bar{k} \rightarrow \frac{\sin n\bar{\mu}\gamma\bar{k}}{n\bar{\mu}\gamma} \quad (\text{A6})$$

in the classical expressions. Here

$$\bar{\mu} = \sqrt{\frac{\Delta}{\bar{p}}} \quad \text{where} \quad \Delta = 2\sqrt{3}\pi\gamma l_{\text{Pl}}^2.$$

This kind of corrections we call holonomy ones. Factor n can be fixed from requirement of the anomaly cancellation [14, 28]. Effective second order Hamiltonian with holonomy corrections takes the form

$$\begin{aligned} H_G^{\text{phen}} &= \frac{1}{16\pi G} \int_{\Sigma} d^3x \bar{N} \left[-6\sqrt{\bar{p}} \left(\frac{\sin \bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma} \right)^2 \right. \\ &\quad - \frac{1}{2\bar{p}^{3/2}} \left(\frac{\sin \bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma} \right)^2 (\delta E_j^c \delta E_k^d \delta_c^k \delta_d^j) \\ &\quad + \sqrt{\bar{p}} (\delta K_c^j \delta K_d^k \delta_k^c \delta_d^j) \\ &\quad - \frac{2}{\sqrt{\bar{p}}} \left(\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma} \right) (\delta E_j^c \delta K_c^j) \\ &\quad \left. + \frac{1}{\bar{p}^{3/2}} (\delta_{cd} \delta^{jk} \delta^{ef} \partial_e E_j^c \partial_f E_k^d) \right] \end{aligned} \quad (\text{A7})$$

where for tensor modes

$$\delta E_i^a = -\frac{1}{2}\bar{p}h_a^a \quad (\text{A8})$$

$$\delta K_a^i = \frac{1}{2} \left[\dot{h}_a^i + \left(\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma} \right) h_a^i \right] \quad (\text{A9})$$

Based on the Hamilton equations

$$\delta \dot{E}_i^a = \left\{ \delta E_i^a, H_G^{\text{phen}} + H_m \right\}, \quad (\text{A10})$$

$$\delta \dot{K}_a^i = \left\{ \delta K_a^i, H_G^{\text{phen}} + H_m \right\}, \quad (\text{A11})$$

we obtain equation

$$\ddot{h}_a^i + 2\bar{k}\dot{h}_a^i - \nabla^2 h_a^i + T_Q h_a^i = 16\pi G \Pi_{Qa}^i \quad (\text{A12})$$

where

$$T_Q = -2 \left(\frac{\bar{p}}{\bar{\mu}} \frac{\partial \bar{\mu}}{\partial \bar{p}} \right) \bar{\mu}^2 \gamma^2 \left(\frac{\sin \bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma} \right)^4, \quad (\text{A13})$$

$$\Pi_{Qa}^i = \left[\frac{1}{3V_0} \frac{\partial \bar{H}_m}{\partial \bar{p}} \left(\frac{\delta E_j^c \delta_a^j \delta_c^i}{\bar{p}} \right) \cos 2\bar{\mu}\gamma\bar{k} + \frac{\delta H_{\eta_a^i}}{\delta (\delta E_i^a)} \right] \quad (\text{A14})$$

are quantum holonomy corrections.

We consider homogeneous scalar field with the Hamiltonian

$$H_m = \int_{\Sigma} d^3x \bar{N} \left(\frac{1}{2} \frac{\pi_{\phi}^2}{\sqrt{\det E}} + \sqrt{|\det E|} V(\phi) \right), \quad (\text{A15})$$

where up to the second order

$$\begin{aligned} \sqrt{\det E} &= \bar{p}^{\frac{3}{2}} \left[1 + \frac{1}{2\bar{p}} \delta_a^i \delta E_i^a - \frac{1}{4\bar{p}^2} \delta_a^i \delta E_j^a \delta_b^j \delta E_i^b \right. \\ &\quad \left. + \frac{1}{8\bar{p}^2} \delta_a^i \delta E_i^a \delta_b^j \delta E_j^b \right], \end{aligned} \quad (\text{A16})$$

$$\begin{aligned} \frac{1}{\sqrt{\det E}} &= \frac{1}{\bar{p}^{\frac{3}{2}}} \left[1 - \frac{1}{2\bar{p}} \delta_a^i \delta E_i^a + \frac{1}{4\bar{p}^2} \delta_a^i \delta E_j^a \delta_b^j \delta E_i^b \right. \\ &\quad \left. + \frac{1}{8\bar{p}^2} \delta_a^i \delta E_i^a \delta_b^j \delta E_j^b \right]. \end{aligned} \quad (\text{A17})$$

However, since $\delta^{ab} h_{ab} = 0 \Rightarrow \delta_a^i \delta E_i^a = 0$ the above expansion simplifies. Then

$$H_m = \bar{H}_m + \frac{1}{4} \int_{\Sigma} d^3x \frac{\bar{N}}{\sqrt{\bar{p}}} \left(\frac{1}{2} \frac{\pi_{\phi}^2}{\bar{p}^3} - V(\phi) \right) \delta_a^i \delta E_j^a \delta_b^j \delta E_i^b + \mathcal{O}(E^3).$$

Now we can derive variation

$$\frac{\delta H_m}{\delta (\delta E_i^a)} = \frac{1}{2} \frac{\bar{N}}{\sqrt{\bar{p}}} \left(\frac{1}{2} \frac{\pi_{\phi}^2}{\bar{p}^3} - V(\phi) \right) \delta_b^i \delta_a^k \delta E_k^b \quad (\text{A18})$$

and derivative

$$\frac{\partial \bar{H}_m}{\partial \bar{p}} = \frac{3}{2} V_0 \frac{\bar{N}}{\sqrt{\bar{p}}} \left(-\frac{1}{2} \frac{\pi_{\phi}^2}{\bar{p}^3} + V(\phi) \right). \quad (\text{A19})$$

One can now easily find that in the classical limit, when we set $\cos(2\bar{\mu}\gamma\bar{k}) = 1$ in expression (A14), the source term vanish. This is due to the opposite signs of the bracketed expression in equations (A18) and (A19). When quantum holonomy corrections are present we have

$$\cos(2\bar{\mu}\gamma\bar{k}) = 1 - 2 \frac{\rho}{\rho_c}, \quad (\text{A20})$$

which can be found from background equations of motion. Therefore the form of the quantum corrections simplifies to

$$T_Q = \frac{8\pi G}{3} \frac{\bar{p}\rho^2}{\rho_c}, \quad (\text{A21})$$

$$\Pi_{Qa}^i = \Pi_Q h_a^i = \frac{1}{2} \bar{p} \frac{\rho}{\rho_c} (2V - \rho) h_a^i, \quad (\text{A22})$$

where we have chosen $\bar{N} = \sqrt{\bar{p}}$ and adopted the expression (A8).

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- [1] M. Bojowald, Living Rev. Rel. **11** (2008) 4.
- [2] Y. Shtanov and V. Sahni, Phys. Lett. B **557** (2003) 1 [arXiv:gr-qc/0208047].
- [3] M. Gasperini and G. Veneziano, Phys. Rept. **373** (2003) 1 [arXiv:hep-th/0207130].
- [4] A. Ashtekar, T. Pawłowski and P. Singh, Phys. Rev. Lett. **96** (2006) 141301 [arXiv:gr-qc/0602086].
- [5] D. Wands, arXiv:0809.4556 [astro-ph].
- [6] Y. F. Cai and X. Zhang, arXiv:0808.2551 [astro-ph].
- [7] Y. F. Cai, T. Qiu, R. Brandenberger, Y. S. Piao and X. Zhang, JCAP **0803** (2008) 013 [arXiv:0711.2187 [hep-th]].
- [8] P. Singh, K. Vandersloot and G. V. Vereshchagin, Phys. Rev. D **74** (2006) 043510 [arXiv:gr-qc/0606032].
- [9] J. Mielczarek, T. Stachowiak and M. Szydłowski, Phys. Rev. D **77** (2008) 123506 [arXiv:0801.0502 [gr-qc]].
- [10] M. Bojowald, H. H. Hernandez, M. Kagan, P. Singh and A. Skirzewski, Phys. Rev. D **74** (2006) 123512 [arXiv:gr-qc/0609057].
- [11] D. Brizuela, G. A. D. Mena Marugán and T. Pawłowski, arXiv:0902.0697 [gr-qc].
- [12] J. Mielczarek and M. Szydłowski, Phys. Lett. B **657** (2007) 20 [arXiv:0705.4449 [gr-qc]].
- [13] J. Mielczarek and M. Szydłowski, arXiv:0710.2742 [gr-qc].
- [14] M. Bojowald and G. M. Hossain, Phys. Rev. D **77** (2008) 023508 [arXiv:0709.2365 [gr-qc]].
- [15] A. Barrau and J. Grain, arXiv:0805.0356 [gr-qc].
- [16] J. Mielczarek, JCAP **0811** (2008) 011 [arXiv:0807.0712 [gr-qc]].
- [17] J. Grain and A. Barrau, arXiv:0902.0145 [gr-qc].
- [18] E. J. Copeland, D. J. Mulryne, N. J. Nunes and M. Shaeri, arXiv:0810.0104 [astro-ph].
- [19] G. Calcagni and G. M. Hossain, arXiv:0810.4330 [gr-qc].
- [20] A. Corichi and P. Singh, Phys. Rev. D **78** (2008) 024034 [arXiv:0805.0136 [gr-qc]].
- [21] J. Mielczarek, arXiv:0809.2469 [gr-qc].
- [22] D. Wands, Phys. Rev. D **60** (1999) 023507 [arXiv:gr-qc/9809062].
- [23] [Planck Collaboration], arXiv:astro-ph/0604069.
- [24] D. Baumann *et al.* [CMBPol Study Team Collaboration], arXiv:0811.3919 [astro-ph].
- [25] C. E. North *et al.*, arXiv:0805.3690 [astro-ph].
- [26] .. P. G. Castro *et al.* [QUaD collaboration], arXiv:0901.0810 [astro-ph.CO].
- [27] D. Samtleben and f. t. Q. collaboration, arXiv:0806.4334 [astro-ph].
- [28] M. Bojowald and G. M. Hossain, Class. Quant. Grav. **24** (2007) 4801 [arXiv:0709.0872 [gr-qc]].